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Peculiarities of the Quanto-Mechanical Space-Time Description

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Abstract

The present paper deals with the analysis of the meaning and of the role of space-time quanta. A consistent mathematical description of the non-transitive binary 'equivalence' is obtained. In connection with the existence of certain threshold velocities some peculiarities of the high-energy behaviour of scattered nucleons are explained.

1. *Introduction*

It has been accepted that physical space and time may be defined only in the presence of microparticles. Under these conditions the microparticles maintain their 'starting' individuality so far as the possibility of performing the 'starting' quanto-mechanical space-time description is assured. Consequently, if a failure within the space-time description arises, some changes in the individuality of the microparticles are implied.

Along the line of the above considerations meaning and role of the space-time quanta are analysed. At low energies, the existence of a possible structure of the microparticle may be neglected since the binding energies of the constituents are larger than their kinetic energies (Kuti, 1971). At high energies this does not follow. In such a situation the space-time quanta—which were defined in connection with the existence of a single 'elementary' particle (Papp, 1971, 1972a, 1972b)—become non-suitable to be attributed to a system of constituents. Otherwise it may be stated that the 'starting' space-time quanta loses—as a result of the kinetic energy excess—their initial reason. Moreover, space-time compatibility ceases to be preserved. Assuming that at high energies the role of a certain structure of the 'elementary' particle ceases to be negligible, the interaction products are generally different at high energies than at low energies. In this sense, the low-energy elastic scattered microparticle is generally substituted, with the increase of energy, by a many microparticle system of interaction products. Consequently, when collision processes of sufficiently high energy are considered, the space-time description of the scattered particle is that one which has to be redefined and set in agreement with the new experimental situation. Conversely, the failure of the space-time description of the elastically scattered particle allows one to suppose the existence of a new experimental situation.

The above considerations will be proved in the case of the scattered nucleons. This is, in fact, the case in which many relevant high-energy data were recently acquired (see for example Kögerler, 1972). We have to mention that the peculiarities of the high-energy behaviour of scattered nucleons were already predicted by Heisenberg (1938).

We shall begin by proposing a mathematical model for the binary description (Kálnay & Toledo, 1967) and for the non-transitive binary 'equivalence'. Scale invariance behaviour, action quantization, space-time operators, and space-time quanta are then analysed. Using some results of space-time compatibility, there is the possibility of predicting some aspects of the high-energy behaviour of the scattered nucleon.

Neglecting the spin of the nucleon we shall describe it by means of a K-G field. Throughout this paper the scattered nucleon is considered in the centre-of-mass system. We shall take $\hbar = c = 1$, except in some self-evident cases.

2. *Binary Description and Binary 'Equivalence'*

As has been proved by Kálnay (1967, 1971) the use of binary variables represents a suitable tool in order to express the result of space-time measurements. The binary variables offer, through their own meaning, the possibility of making explicit the imprecision boundary of the measurement (Papp, 1971, 1972).

The use of the binary variables implies apparent peculiarities so that the mathematical description and the interpretative formalism must be adequately defined. In this respect, taking into consideration the binary evaluation $\alpha - i\beta$, where $\beta > 0$, we may propose, in order to define the measurable meaning, the following mathematics:

(a) The binary variable $\alpha - i\beta$ describes the measurement in which the result α within the imprecision β is obtained. In agreement with Kálnay & Toledo (1967) the binary variable defines the segment $[\alpha - \beta, \alpha + \beta]$ on the α-axis. This segment also has to be considered to express the result of the measurement.
(b) If $|\alpha| \leqslant \beta$ the binary evaluation does not possess measurable meaning.
(c) If $\beta < |\alpha| < 3\beta$ we have to consider that its measurable meaning is undetermined.

(d) If $|\alpha| \geqslant 3\beta$ the binary evaluation possesses a well-defined measurable meaning.

(e) Two binary evaluations $\alpha_1 - i\beta$ and $\alpha_2 - i\beta$ are binarily 'equivalent' if

$$|\alpha_1 - \alpha_2| \leqslant 2\beta \tag{2.1}$$

If two binary evaluations are binarily 'equivalent' we shall consider that they refer to the same measurement, i.e. to the same experimental situation.

We remark that point (c) avoids binary 'equivalence' between binary evaluations which possess and those which do not possess (a well-defined) measurable meaning. Thus, the role of point (c) is well established. One can easily observe, in agreement also with March (1941), that the binary 'equivalence' generally does not satisfy the transitivity property. Indeed, if $|\alpha_1 - \alpha_2| \leqslant 2\beta$ and $|\alpha_2 - \alpha_3| \leqslant 2\beta$, the inequality $|\alpha_1 - \alpha_3| \leqslant 2\beta$ is not necessarily implied. In this sense, the non-transitivity of the binary 'equivalence' may be considered as a representative peculiarity of the quanto-mechanical description of microparticles.

(f) We now have to analyse the conditions in which two binary evaluations, which do not possess the same imaginary part, may be considered as binarily 'equivalent'. In this respect we may assume that the result of a measurement given by $\alpha - 2i\beta$ may be expressed under the form of the set:

$$\mathfrak{M}^{(2)} \equiv \{\alpha - \beta - i\beta,\ \alpha - i\beta,\ \alpha + \beta - i\beta\} \tag{2.2}$$

The set $\mathfrak{M}^{(2)}$ and the binary evaluation $\alpha - 2i\beta$ define on the α-axis the same segment $[\alpha - 2\beta, \alpha + 2\beta]$. The result of the measurement being a region in space, both the set $\mathfrak{M}^{(2)}$ and $\alpha - 2i\beta$ give a description of the same measurement. In this respect one needs to mention that the binary evaluations $\alpha - \beta - i\beta$, $\alpha - i\beta$ and $\alpha + \beta - i\beta$ can describe the same measurement as they are binarily 'equivalent'.

Considering the binary evaluation $\alpha - in\beta$, where (for instance) n is an integer larger than 2, we may formally decompose it in the form of the set

$$\mathfrak{M}^{(n)} \equiv \{\alpha - (n-1)\beta - i\beta, \ldots, \alpha + (n-1)\beta - i\beta\} \tag{2.3}$$

It may be verified that between the elements of the set $\mathfrak{M}^{(n)}$ there exists a number

$$D_n = 2n^2 - 7n + 6 \tag{2.4}$$

of relations of binary 'non-equivalence'. Consequently—excepting the cases in which $n = \frac{3}{2}$ and $n = 2$ (when $D_n = 0$)—the elements of the set $\mathfrak{M}^{(n)}$ do not refer to the same measurement. In these conditions it may be considered that the binary evaluations $\alpha - i\beta_1$ and $\alpha - i\beta_2$

where $\beta_2 > \beta_1$ are binarily 'equivalent' only if $\beta_2 = \frac{3}{2}\beta_1$ or $\beta_2 = 2\beta_1$. However, generalizing the above result, we may define—at least as an example—an extended binary 'equivalence' relation requiring that $\beta_2 \leqslant 2\beta_1$. The above statements do not conflict with the conditions (b)–(d).

(g) The binary evaluations $\alpha - i\beta$ and $\alpha + i\beta$ are 'identical' as the same imprecision β refers to the same 'observable' evaluation α.

Particularly, the evaluations $3\beta - i\beta$ and $5\beta - i\beta$ are binarily 'equivalent' and possess a well-defined measurable meaning. Consequently, the most representative 'observable' evaluation, corresponding to the first 'equivalence class' of meaningful binary variables, is given by 4β.

3. *Scale Invariance Behaviour*

Let us consider the scale invariant field

$$\lambda^d \Phi(\lambda x) = U^*(\lambda)\, \Phi(x)\, U(\lambda) \tag{3.1}$$

where $U(\lambda)$ is an unitary operator, $\lambda = 1 - \varepsilon$ $(0 \leqslant \varepsilon < 1)$, and where d gives the dimension of the field (Wilson, 1970).

Considering a K-G field and supposing that the scale invariance behaviour is exclusively supported by the rest mass m_0:

$$m_0 \to m_0' = \lambda^{-1} m_0 \simeq m_0 + \varepsilon m_0 \tag{3.2}$$

so that

$$(\mathbf{p}, p_0) \to (\mathbf{p}', p_0') = \lambda^{-1}(\mathbf{p}, p_0) \tag{3.3}$$

one obtains, using the usual expansion (Bjorken & Drell, 1965), the result that the annihilation operator $a^{(+)}(\mathbf{p})$ transforms as follows:

$$\lambda^{d-5/2}\, a^{(+)}(\lambda^{-1}\mathbf{p}) = U^*(\lambda)\, a^{(+)}(\mathbf{p})\, U(\lambda) \tag{3.4}$$

At a well-defined value of the angular momentum the above relation becomes

$$\lambda^{d-3/2}\, a_l^{(+)}(\lambda^{-1} p) = U^*(\lambda)\, a_l^{(+)}(p)\, U(\lambda) \tag{3.5}$$

where $p \equiv |\mathbf{p}|$ and where a rotation symmetry of the annihilation operator around a certain axis has been assumed.

In this latter case

$$a^{(+)}(\mathbf{p}) = p^{-1} \sum_l Y_{l,0}(\operatorname{vers}\mathbf{p} \,.\, \operatorname{vers}\mathbf{k})\, a_l^{(+)}(p) \tag{3.6}$$

where vers$\mathbf{k}$ expresses the unit vector of the symmetry axis. Setting $d = \frac{3}{2}$, the scale invariance becomes a symmetry operation.

We may thus conclude—in agreement with (3.2)—that the existence of a particle production process is able to support the existence of the scale invariance behaviour. In this respect some calculations may be performed approximately.

Indeed, the rest mass increase, corresponding to a certain kinetic energy Δp_0, is approximately given by

$$\Delta m_0 \simeq \frac{2m_0 p_0}{p^2} \Delta p_0 = \frac{2m_0}{v(1-v^2)} \Delta v \tag{3.7}$$

In agreement with the space-momentum uncertainty relation we may consider that the 'minimum' value of the velocity uncertainty Δv is given by

$$\Delta v_m = \frac{\hbar}{2m_0 \Delta x}(1-v^2)^{3/2} \tag{3.8}$$

where $\Delta x \approx 10^{-13}$ cm represents the interaction radius of strong interacting particles. Then we will obtain around $v \approx 0{\cdot}6c$, $\Delta v_m \approx 0{\cdot}06c$ so that $\Delta m_0 \approx 150$ MeV. But this rest-mass increase expresses an acceptable approximation of the pionic rest mass. In these conditions we may consider that, in the case of the scattered nucleon, the kinetic energy excess implies a single pion production. Such a process can arise as soon as the scattered nucleon takes—in the approximation considered—an energy around 1200 MeV. This energy is in reasonable agreement with the results concerning the excitation of the nucleonic N^*-resonance (see for example Schultze, 1971; Zucker, 1971).

Thus, at the threshold of the single-pion production, the parameter ε has to take approximately the value

$$\varepsilon \simeq \frac{\Delta m_0}{m_0} \simeq \frac{2}{\Delta x}\left\langle\frac{1}{2p}\right\rangle_l = \frac{2}{\Delta x}\delta_l^{(+)}\tilde{s} \tag{3.9}$$

where a well-defined value of the angular momentum has been assumed and where the quantity $\delta_l^{(+)}\tilde{s}$ expresses the total space quantum. By means of the above relation we may assume that the nucleon constituents which are nearer to the nucleon 'centre' must possess a larger rest mass than the pion, so that—at sufficiently large energies—the emission of the heavy constituents is also expected. In these conditions the problem to define the hadronic mass spectrum is formally reduced to the defining problem of the ε-spectrum, i.e. of the $\delta_l^{(+)}\tilde{s}$-spectrum.

4. *Peculiarities of the Quantum-mechanical Action*

The scale transformation of the one-particle amplitude

$$g_l(p) = \langle 0|a_l^{(+)}(p)|\psi\rangle \tag{4.1}$$

where $|\psi\rangle$ expresses the general state of the K-G field, is given in agreement with (3.5) by

$$g_l(\lambda^{-1}p) = \langle 0|a_l^{(+)}(p)\, U(\lambda)|\psi\rangle \equiv U_p(\lambda)\, g_l(p) \tag{4.2}$$

where $d = \frac{3}{2}$. The operator $U_p(\lambda)$ so introduced expresses the quantum-mechanical unitary operator of the scale transformation.

Similarly, the one-particle amplitude in the energy representation transforms as

$$\tilde{g}_l(\lambda^{-1} p_0) = \langle 0|\tilde{a}_l^{(+)}(p_0)\, U(\lambda)|\psi\rangle \equiv U_{p_0}(\lambda)\, \tilde{g}_l(p_0) \tag{4.3}$$

where $\tilde{a}_l^{(+)}(p_0) = \sqrt{(p_0/p)}\, a_l^{(+)}(p)$. On the other hand—irrespective of (3.5)—the relations

$$g_l(\lambda^{-1} p) = \exp\left[-\ln \lambda p \frac{\partial}{\partial p}\right] g_l(p) \tag{4.4}$$

and also

$$\tilde{g}_l(\lambda^{-1} p_0) = \exp\left[-\ln \lambda p_0 \frac{\partial}{\partial p_0}\right] \tilde{g}_l(p_0) \tag{4.5}$$

are valid. Consequently, up to the factor λ^{-1}, the pr-action operator and the $p_0 t$-action operator are stated as Hermitian observables in the p-momentum and respectively in the energy representation. Using the relations (4.4) and (4.5) the unitary operators $U_p(\lambda)$ and respectively $U_{p_0}(\lambda)$ may be easily identified.

Passing from the p-momentum representation to the energy representation, the Hermitian space operator $i(d/dp)$ becomes a binary operator (Papp, 1971, 1972a). In these conditions we cannot pass from the p-momentum representation to the energy representation without implying the existence of the real space quantum $\frac{1}{2}\langle m_0^2/pp_0^2\rangle$.

Consequently, in spite of the fact that the relations (4.2) and (4.3) are both 'equivalent' to the relation (3.5), there is no 'equivalence' between (4.2) and (4.3), because within the energy representation the existence of the real space quantum cannot be ignored.

Ignoring the existence of the space quantum and imposing the requirement that (4.2) and (4.3) have to be 'equivalent', rather unphysical results would be obtained.

Indeed, using the approximations

$$U_p(\lambda) \simeq 1 + \varepsilon p \frac{\partial}{\partial p}, \qquad U_{p_0}(\lambda) \simeq 1 + \varepsilon p_0 \frac{\partial}{\partial p_0} \tag{4.6}$$

the relations (4.4) and (4.5) become 'equivalent' only if

$$\left(\frac{p_0^2}{p^2}\frac{d}{dp} - \frac{d}{dp} - \frac{m_0^2}{2p_0^2 p}\right) \tilde{g}_l(p_0) = 0 \tag{4.7}$$

where the explicit existence of the real space quantum may be observed. In these conditions

$$\tilde{g}_l(p_0) = \tfrac{1}{2} \ln p_0 + \text{const} \tag{4.8}$$

and such a function is not able to perform a quantum-mechanical wave-packet description of a single K-G particle because of its divergence at infinity. Conversely there is no physical object which can be described by such a divergent function.

Consequently, within a meaningful physical description performed in the energy representation, the existence of the real space quantum cannot be neglected. Similar results will be obtained if the 'equivalence' of the relations (4.2) and (4.3) is tested passing from the energy representation to that of the p-momentum. In this latter case the existence of the real time quantum cannot be ignored.

5. *Space-Time Quanta and Space-Time Operators*

Within the interpretative formalism of the binary variables the space and time quanta have to be identified with the imaginary parts of the corresponding binary entities. The quanta so obtained are—excepting the proper time quantum (Papp, 1972b)—observer dependent (Papp, 1972a).

In these conditions the imaginary part of the usual binary time is given by the so-called real time quantum

$$\delta_l^{(+)}\tau = \frac{1}{2}\left\langle\frac{m_0^2}{p^2 p_0}\right\rangle_l = \frac{1}{2}\left\langle\frac{p_0}{p^2}\right\rangle_l - \frac{1}{2}\left\langle\frac{1}{p_0}\right\rangle_l \tag{5.1}$$

Assuming the imprecision additivity it follows that there also exists the quantum

$$\delta_l^{(+)}\tilde{\tau} = \frac{1}{2}\left\langle\frac{p_0}{p^2}\right\rangle_l \tag{5.2}$$

which, being larger than $\delta_l^{(+)}\tau$, may be considered as the total time quantum.

As a consequence of the existence of the above time quanta the real space quantum

$$\delta_l^{(+)}s = \frac{1}{2}\left\langle\frac{m_0^2}{p p_0^2}\right\rangle_l = \frac{1}{2}\left\langle\frac{1}{p}\right\rangle_l - \frac{1}{2}\left\langle\frac{p}{p_0^2}\right\rangle_l \tag{5.3}$$

exists and also the total space quantum

$$\delta_l^{(+)}\tilde{s} = \frac{1}{2}\left\langle\frac{1}{p}\right\rangle_l \tag{5.4}$$

The imprecisions so defined result from performing the averages of certain space-time operators with respect to the single particle amplitude $g_l(p)$ with the conditions

$$\lim_{p\to\infty} g_l(p) = 0, \qquad \lim_{p\to 0} p^{-1/2} g_l(p) = 0 \tag{5.5}$$

Taking into consideration the p-momentum representation, it may be proved that, besides the binary time operator $i(d/dp_0)$ leading to the imprecision $\delta_l^{(+)}\tau$ there exists the total time-operator

$$\tilde{T} = i\frac{d}{dp_0} + i\frac{1}{2p_0} \tag{5.6}$$

corresponding to the total time imprecision.

The binary space operator then takes the form

$$S_{(b)} = i\frac{d}{dp} + i\frac{m_0^2}{2pp_0^2} \tag{5.7}$$

whereas the total space operator is given by

$$\tilde{S} = i\frac{d}{dp} - i\frac{p}{2p_0^2} \tag{5.8}$$

In connection with the above results it may be mentioned that the condition (5.5) supports not only the existence of the binary space-time operators, but also the existence of the Hermitian time operator

$$T^{(*)} = i\frac{d}{dp_0} - i\frac{m_0^2}{2p^2 p_0} \tag{5.9}$$

Consequently no binary space-time operators may be defined without assuming the fulfilment of the formal hermiticity condition of the time operator. In this way the binary operators may be stated as 'natural' extensions of the Hermitian observables; such operators preserve (effectively) the 'observability' property as soon as the boundary condition (5.5) is fulfilled.

The real space-time operators and the total space-time operators are binarily 'equivalent' in the extended sense, up to the mean velocity $[\sqrt{(2)}/2]c$. Denoting with v_0 the mean velocity we shall obtain:

$$\begin{aligned} \delta_i^{(+)} \tilde{s} &\leqslant 2\delta_i^{(+)} s \\ \delta_i^{(+)} \tilde{\tau} &\leqslant 2\delta_i^{(+)} \tau \end{aligned} \tag{5.10}$$

for $v_0 \leqslant [\sqrt{(2)}/2]c$. Consequently, up to the mean velocity $[\sqrt{(2)}/2]c$ the real and the total space-time operators do not possess a separate measurable meaning.

At a well-defined value of the angular momentum the total space operator $\tilde{S}$ possesses formally the same form as the previously defined Newton–Wigner position operator (Newton & Wigner, 1949). However, these operators are identical only in the presence of the boundary condition assuring the existence of the hermitic time operator $T^{(*)}$.

We may consider that all the space and time quanta preserve their meaning only in respect of the existence of a certain lower bound. Conversely, with increasing velocity, the space and time quanta become as small as possible, so that the measurements become as exact as possible. Such a result would contradict the quantum-mechanical significance of the space-time measurements. On the other hand the imprecision of the space-time measurements cannot be indefinitely decreased without affecting the existence of the system submitted to the measurement. Consequently the lower bound imposed to the space and time quanta has also to possess the meaning of a structural constant of the system. Such a constant must be observer independent. Respecting the above considerations we may suppose the time constant to be identical with the proper time quantum (Papp, 1972b)

$$\delta^c \tau = \frac{\hbar}{2m_0 c^2} \tag{5.11}$$

The space-constant should be given by

$$\delta^c s = \frac{\hbar}{2m_0 c} \tag{5.12}$$

We may easily calculate that the above space and time constant are able to fulfil their limiting role respecting the space and time quanta. They also have the significance of natural space and time units (March, 1941). Indeed

$$\delta_i^{(+)} s \geqslant \delta^c s, \qquad v_0 \leqslant v_s \simeq 0{\cdot}564c \tag{5.13}$$

$$\delta_i^{(+)} \tau \geqslant \delta^c \tau, \qquad v_0 \leqslant v_\tau \simeq 0{\cdot}656c \tag{5.14}$$

$$\delta_i^{(+)} \tilde{s} \geqslant \delta^c s, \qquad v_0 \leqslant v_{\tilde{s}} = \frac{\sqrt{2}}{2} c \simeq 0{\cdot}707c \tag{5.15}$$

$$\delta_i^{(+)} \tilde{\tau} \geqslant \delta^c \tau, \qquad v_0 \leqslant v_{\tilde{\tau}} = \sqrt{\frac{\sqrt{(5)} - 1}{2}}\, c \simeq 0{\cdot}786c \tag{5.16}$$

where the velocity on the right expresses the upper velocity value for which the inequality on the left is fulfilled.

In these conditions the products of space and time quanta are also limited by the products of the space and time constants:

$$\delta_i^{(+)} s\, \delta_i^{(+)} \tau \geqslant \delta^c \tau\, \delta^c s, \qquad v_0 \leqslant v_{(1)} = \frac{\sqrt{(5)} - 1}{2} c \simeq 0{\cdot}618c \tag{5.17}$$

$$\delta_i^{(+)} s\, \delta_i^{(+)} \tilde{\tau} (\simeq \delta_i^{(+)} \tilde{s}\, \delta_i^{(+)} \tau) \geqslant \delta^c \tau\, \delta^c s, \qquad v_0 \leqslant v_{(2)} \simeq 0{\cdot}671c \tag{5.18}$$

$$\delta_i^{(+)} \tilde{s}\, \delta_i^{(+)} \tilde{\tau} \geqslant \delta^c \tau\, \delta^c s, \qquad v_0 \leqslant v_{(3)} \simeq 0{\cdot}755c \tag{5.19}$$

For instance, seven presumptive threshold velocities are implied. The existence of a set of threshold velocities may be considered as a result of using binary variables. However, there is the possibility of analysing the set in terms of the limitations involved by the space-time compatibility condition.

6. *Space-Time Compatibility*

All the previous results concerning space-time quantization are valid—because time is defined respecting space—only in the conditions in which space-time compatibility is assured. The space-time compatibility may be analysed either for non-zero values of the macroscopic time parameter t, or for $t=0$. In the first case it has been proved—using field-theoretical methods—that the compatibility is fulfilled attributing to the t-parameter a binary meaning (Papp, 1973). In the second case there are implied other restrictions concerning validity and meaning of space-time quantization.

Assuming space hermiticity the imprecision of the space-time measurement is taken over by time. In this respect the Hermitian space operator $i(d/dp)$ is compatible with the binary time $i(d/dp_0)$ in the conditions in which the commutator

$$\left[\frac{d}{dp},\frac{d}{dp_0}\right]=-m_0{}^2p^{-2}p_0^{-1}\frac{d}{dp} \tag{6.1}$$

does not possess measurable meaning.

Performing the calculations it may be proved that

$$\left\langle\frac{m_0{}^2}{p^2p_0}\frac{d}{dp}\right\rangle_l=i\left\langle\frac{m_0{}^2}{p^2p_0}\frac{d}{dp}\arg g_l(p)\right\rangle_l+\left\langle\frac{m_0{}^2}{p^3p_0}+\frac{m_0{}^2}{2pp_0{}^3}\right\rangle_l \tag{6.2}$$

so that the condition (b) of Section 2 is fulfilled if

$$v_0\leqslant f(v_0)\equiv\sqrt{\frac{N(v_0)-2}{N(v_0)+1}}\,c \tag{6.3}$$

where the function $N(v_0)$ satisfies the equality

$$\frac{d}{d\langle p\rangle_l}\arg g_l(\langle p\rangle_l)=N(v_0)\,\delta_l^{(+)}s \tag{6.4}$$

For $N\geqslant 2$ the f-function takes real values and as $df/dN>0$ it is an increasing function respecting N. In these conditions a lower bound of the f-function may be defined: taking into consideration only the well-measurable space-shifts we have to take $N(v_0)\geqslant 4$ (see Section 2). In this case

$$f(v_0)\geqslant\sqrt{(\tfrac{2}{5})}\,c\equiv v_c\simeq 0{\cdot}632c \tag{6.5}$$

This threshold velocity expresses the upper velocity for which—respecting well-measurable space-time evaluations—space-time compatibility is always assured. This fact means that, in the conditions in which for example the energy of the scattered nucleon overpasses $p_0(v_c)\simeq 1212$ MeV,† the existing space-time description loses its reason for existence.

The above results may also be obtained if we would assume that, in the high-energy domain (around $v_0\approx v_c\approx v_\tau$), the space shift of the scattered nucleon behaves as

$$\frac{d}{d\langle p\rangle_l}\arg g_l(\langle p\rangle_l)\simeq N\delta_l^{(+)}s \tag{6.6}$$

† Performing numerical calculations we shall consider for simplicity only the rest mass of the neutron.

where N actually expresses a parameter not depending on velocity. The above inequality also expresses the manner in which the real space quantum (and more exactly the real time quantum) realises its role of a natural space unit. In this situation, using (6.6) we can analyse some physical aspects of the high-energy behaviour of the scattered nucleon.

Neglecting the explicit influence of the measuring apparatus, the scattering phase shift $\delta_l(\langle p\rangle_l)$ may be identified with $\arg g_l(\langle p\rangle_l)$. Integrating (6.6) between p and infinity gives, up to the sign, the result that

$$\delta_l(p) = \tfrac{1}{2}N \ln \frac{p}{p_0} \tag{6.7}$$

where the $\langle p\rangle_l$-average has been replaced by p. Consequently the S-matrix approximately takes the form

$$S_l(p) \simeq \frac{p - \frac{i}{2}N(p_0 - p)}{p + \frac{i}{2}N(p_0 - p)} \tag{6.8}$$

for $v_0 \geqslant \frac{1}{2}c$. For this purpose the approximation $\ln x \simeq (x-1)/x$ valid for $x \geqslant \frac{1}{2}$ and the approximation $\mathrm{tg}\,\delta_l \simeq \delta_l$ have been used. A more suitable form of the S-matrix may be obtained also by using—in the high-energy domain—the approximation

$$p_0 \simeq p + \frac{{m_0}^2}{2p} \tag{6.9}$$

which is mathematically valid for $v > [\sqrt{(2)}/2]c$. In the following we shall consider the expression (6.9) as a reasonable physical approximation to the energy for velocities which are, within the uncertainty bound, smaller than $[\sqrt{(2)}/2]c$ too. In these conditions the S-matrix becomes

$$S_l(p) \simeq \frac{p^2 - i\frac{N}{4}{m_0}^2}{p^2 + i\frac{N}{4}{m_0}^2} \tag{6.10}$$

Thus, the high-energy behaviour (6.6), assuring the existence of the threshold velocity v_c, implies the existence of two S-matrix poles

$$p_c^{(\pm)} = \pm m_0 \frac{\sqrt{2N}}{4}(1-i) \tag{6.11}$$

placed in the second and fourth quadrant respectively of the complex p-plane. This fact signifies that, at high energies, there should exist, within the considered approximation, a superposition of resonance production—

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described by the pole $p_c^{(-)}$—and of resonance decay—described by the pole $p_c^{(+)}$. But such behaviour in fact expresses a particle production process, so that the previous suppositions concerning the high-energy behaviour of the scattered nucleon, are, at least, qualitatively confirmed. Using again $p_0 = \sqrt{(\mathbf{p}^2 + m_0{}^2)}$ the energy and width of the resonance so implied are given by

$$p_0^{\mathrm{res}} \simeq 1227 \text{ MeV}, \qquad \frac{\Gamma}{2} \simeq 323 \text{ MeV} \tag{6.12}$$

and this result is in relatively good agreement with that one obtained for the (weak) excitation of the N^*-resonance. In this respect the interpretation from Section 3 concerning single-pion production is correct too, since it was experimentally proved that the cross-section for single-pion production is well approximated by the cross section of the N^*-resonance excitation (Schultze, 1971).

Calculating the energies corresponding to the previous threshold velocities one obtains energies near, or not much larger, than that of the N^*-resonance. The nearest energy to that of the N^*-resonance is that of 1235 MeV corresponding to the threshold velocity v_τ. Summarising the previous results we may then consider that the threshold velocity v_τ fulfills the following roles: (a) it is the velocity at which the space-time quanta realise their role as natural space-time units; (b) it is the threshold of resonance pionic production; (c) it is the upper velocity assuming compatibility of the binary time and Hermitian space.

Using (6.8) and taking $N = 4$ it may similarly be proved that the compatibility between the binary space and total time, the binary time and total space, the total space and total time is assured up to the threshold velocities $0{\cdot}79c$; $0{\cdot}65c$ and $0{\cdot}61c$, respectively. We have to mention that, in the conditions in which space hermiticity is not assumed, the compatibility between binary space and binary time (i.e. the compatibility between the operators whose imprecisions are given by the real space quantum and real time quantum respectively) is assured as soon as $N \geqslant 1$, without the presence of any threshold velocity.

Consequently, in connection with the existence of certain structural effects, a certain physical meaning may be attributed to the involved space-time quantum and/or to the corresponding threshold velocity. In this respect—in agreement with the above interpretations—a 'hadronic' meaning may be attributed to the real time quantum and also to the corresponding threshold velocity.

Similarly, taking into consideration the remarks expressed by Jaffe & Shapiro (1972), we are able to attribute in a certain sense an 'electromagnetic' meaning to the total space quantum and to the corresponding threshold velocity of $[\sqrt{(2)}/2]c$. However, further investigations are needed in order to clarify and to explain the physical meaning of the threshold velocities previously introduced.

7. *Conclusions*

In this paper the meaning and the role of the space-time quanta have been analysed. In this respect it has been proved that there is the existence of space and time quanta that makes 'equivalence' relations to be non-transitive and vice versa.

The space-time quantization may be used in order to perform not only qualitative, but even quantitative predictions concerning the high-energy behaviour of the scattered nucleon. The existence of threshold velocities—which have the role of preserving the initial individuality of the quanto-mechanical system—has to be taken into account. Beyond threshold velocity, particle production is implied. Consequently we have to perform a new space-time description suitable to the changed experimental situation.

With the increase of the energy the space-time quanta tend to realise their role of natural space-time units. This role once fulfilled, the physical system, i.e. the scattered nucleon, ceases to exist within the initial structure constants and also within the initial space-time description.

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